# COMPUTER SIMULATION AND EXPERIMENTAL STUDY FOR CALCULATION OF SLAB SLIP FORWARD FACTOR IN FINISHING ROLL AREA OF HOT STRIP MILL OF MOBARAKEH STEEL COMPANY 

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#### Abstract

Slip forward factor of work-piece is one fundamental rolling parameters and its accurate calculation makes the calculation and determination of friction factor and accurate calculation of roll force possible. By knowing the state of friction of the rollers, one can prevent skidding which causes inappropriate deformations of the work-piece. In consecutive rolling stands to obtain the start up motor speed, first, the speed of work-piece in stand output is calculated and then by calculating the slip forward factor of work-piece in that stand, the linear speed of the roller will be calculated and then angular speed of the main motor of the stand will be determined. Therefore, calculating rollers' speed and their motors necessitates calculation of this factor. In this article, an effective method for measuring the slip forward factor in finishing rolling stand will be explained. Slip forward factor in each of finishing rolling stands is calculated through an empirical formula. This empirical formula contains some constant coefficients which in this research will be calculated by an identification method.


Keywords: slip forward; adaptation model; Mobarakeh steel company; finishing roll; friction

## INTRODUCTION

The rolling industry is the most generic and widespread method of producing metal products, especially for steels. Nowadays, more than $80 \%$ of metal products around the globe are produced by this method. The rolling process consists of deforming metals by cold rolling (without heating the metal) or hot rolling (with heating the metal over $1000^{\circ} \mathrm{c}$ ).

The process of hot rolling consists of passing hot metals between two rollers which rotate in opposite directions and the distance between them is less than the thickness of the piece of metal. Due to this, the metal will be pressed and plastic deformation occurs; therefore, the work-piece elongates and it cross section will change, [1-10].

One of the main sections of the hot strip mill which actually is its heart is the finishing mill whose main responsibility is to decrease the thickness of output load from the primary mill area and changing it to the finishing production, i.e., the sheet. In the finishing mill section of Mobarakeh Steel Company, seven rolling stands exist whose duty is to gradually decrease thickness of the load and providing the finished product. To prevent excessive defonmation of the rollers and accurate control of the thickness and width of the load in the stands of primary and finishing mills, four working rollers and two supporting rollers are used.

Due to their duty, all of the hot strip mill equipments have one or more variable parameters which should be adjusted to their amounts (set-points) for quality production. The main set-points of ultimate rolling pin equipments area are:

1. Gap of each lateral directive.
2. The short course of area lateral directive.
3. Gap of each rolling stands.
4. Rollers' speed of each stands.
5. Amount of load extension load between each two rolling consecutive stands.
6. The additional amount of material between each two consecutive rolling stands.

In consecutive rolhing stands, first, speed of piece of work at the stand output is calculated in order to find the speed of startup motors of rolling stands. Then, by obtaining the slip forward factor in each stand, the linear speed of the roller is calculated and next their angular speed is determined and finally, the angular speed of the main start up motor of each stand is calculated. Therefore, calculating the roller speed and their motors requires calculating this factor.

In this article, an efficient method to measure the slip forward factor in ultimate rolling stands of hot strip mill area at Mobacakeh Steel Company will be explained. Slip forward factor in each ultimate rolling stand is calculated through an empirical formula. This empirical formula contains some coefficients, which in this study area determined through and identification method.

## CALCULATIO OF SLIP FORWARD FACTOR IN FINISSING ROLL STANDS

Slip forward factor of piece of work is one of the fundamental rolling parameters and its accurate calculation, makes the calculation and determination of friction factor and accurate calculation of roll force possible. By knowing the shape of friction of the rollers, skidding which causes nonuniform deformation of the piece of work can be prevented. At the present time, in calculating model of ultimate rollers speed of hot strip mill area of Mobarakeh Steel Company, which is
designed by Siemens Company of Germany, calculation of slip forward in rolling pin stands is carried out using the following formula:

$$
\begin{equation*}
\mathrm{F} . \mathrm{S}=\frac{\mathrm{V}_{1}-\mathrm{V}_{\mathrm{r}}}{\mathrm{~V}_{\mathrm{r}}}=\mathrm{f}\left(\sqrt{\frac{\mathrm{R}}{\mathrm{~h}_{0}}}\right) \cdot \mathrm{g}(\operatorname{Red}) \tag{1}
\end{equation*}
$$

In which, $V_{I}$ is the speed of output work-piece of the rolling stands, $V_{r}$ is the linear speed of the rollers, $R$ is the roller radius, $h_{0}$ is the thickness of input work-piece to the stand and Red is the relative thickness reduction of the work-piece in the rolling stand. Also $f$ and $g$ are polynomials which are assumed to be as follows:

$$
\begin{aligned}
& f(x)=a_{0}+a_{1} x+a_{2} x^{2}+a_{3} x^{3}+a_{4} x^{4}+a_{5} x^{5} \\
& g(y)=b_{0}+b_{1} y+b_{2} y^{2}+b_{3} y^{3}+b_{4} y^{4}+b_{5} y^{5}
\end{aligned}
$$

In the above polynomials $a_{0}$ to $a_{5}$ and $b_{0}$ to $b_{5}$ are constant coefficients which depend on the type of rollers, the amount of extension force between the work-prece and stand, the process of cooling the rollers and the type of friction existing between the rollers and piece of work.
The aforementioned coefficients, in onset of building the hot strip mill area of Mobarakeh Steel Company, have been specified by the designers of this areas ealcullation models, i.e., Siemens Company of Germany as follows:

$$
\left\{\begin{array}{lll}
a_{0}=2.1879 \times 10^{-1} & a_{1}=3.1075 \times 10^{-1} & a_{2}=-5.624 \times 10^{-2}  \tag{4}\\
a_{3}=5.3778 \times 10^{-3} & a_{4}=-2.552 \times 10^{-4} & a_{5}=-4.71 \times 10^{-6}
\end{array}\right.
$$

These coefficients have been determined according to the conditions of the hot strip mill line at setup time. But; from the beginning of hot strip mill till new many changed and modification have been implemented in this area which resulte in modification the slip forward factor of the workpie

Of the most important of these changes, one can point to the addition of stand F7in ultimate rolling section, change of type and dimensions of working and supporting rollers of some of ultimate stands, mounting between stands cooling systems and change of between stands extension force.

Thus, for accurate calculation of slip forward factor in finishing rolling stands and then increased quality of the ultimate product, it is necessary that the formula presented to calculate this factor is put under close investigation and the optimum coefficients current conditions of finishing mill area matching be calculated.

## MEASURING SLIP FORWARD FACTOR IN ROLLING PIN STANDS

In the common and practical methods of measuring slip forward factor in rolling stands, one shaft encoder is used for measuring the speed of the upper and lower rollers the speed of the output
work-piece is usually calculated through measuring the time of the work-piece motion between two photodiode switches or measuring the time of work-piece motion between two consecutive stands. Fortunately, in hot strip mill area of Mobaraked Steel Company, The existing data logger in finishing mill section records many variables and provides graphs of the basic parameters. Therefore, measuring the slip forward factor of each of finishing mill stand is now possible.

As was mentioned in the previous part, the slip forward factor of the work-piece in a rolling stand is defined as the relative difference between work-piece of speed and speed of the working rollers is calculated by this following formula:

$$
\begin{equation*}
\mathrm{F} . \mathrm{S}=\frac{\mathrm{V}_{\text {Out }}-\mathrm{V}_{\mathrm{R}}}{\mathrm{~V}_{\mathrm{R}}} \tag{5}
\end{equation*}
$$


where $V_{\text {out }}$ is speed of output work-piece between
rollers and $V$ rollers.

Hence, to measure the slip forward factor of work-piece in a rolling stand it is necessary that linear speed of the rollers and speed of output work-piece be measured.

The graph of working rollers' speed of each finishing stand is recorded in data logger computer of this area. To measure speed of output work-piece, it is sufficient that the consumed time for the load to pass the distance between the current and the nextstands be measured. By knowing the distance between two consecutive stands work-piece ( 5.5 m ), the speed of output work-piece of each stand is calculated from the following formula:


$$
\begin{equation*}
V_{\text {Out }}=\frac{L}{t} \tag{6}
\end{equation*}
$$

where $L$ is the distance between the current stand and the next stand and $t$ is the load motion time between the two stands.

To measure the motion time of the load between the two stands, stand load signals in data logger computer of the finishing mitt section can be used. Fig. 1 which is taken from the finishing mill section data logger computer illustrates the speed graph of some of the rolling stands and the entrance time of the load to this stands.


Figure 1. Speed graphs of some rolling stands. Thus, slip forward factor in this rolling pin stand can be calculated through measuring the motion time of the output load from one stand to the next stand and measuring the average speed of stand rollers at this period of time (Fig. 2).


## Fig. 2. Measuring the motion time of the output load from one stand to the next. <br> CALGULATION OF OPTIMUM COEFFICIENTS FOR SLIP FORWARD FACTOR FORMULA

In the previous section, a practical method for measuring slip forward factor in finishing mill stands was explained. But, due to some limitations, online measuring of the parameter is not possible for all of the products rolling in the finishing mill section. Therefore, slip forward factor of each of the finishing stands is calculating from an empirical formula (1).

As was explained before, this empirical formula has some constant coefficients which because of changes made in the finishing equipment need review and redetermination. One of the best ways to identify optimum coefficients of the mentioned formula is to use experimental of finish rolling stands.

Unknown parameters of the system can be identified through the determining the input(s) and measuring its output(s). The inputs of this system are as follows: radius of working rollers $(R)$, thickness of the input load to the stand $\left(h_{0}\right)$, and the amount of relative thickness reduction of the load in rolling stand (Red).

The output of the system is, slip forward factor in rolling stands (F.S). The coefficients of formula (2) and (3) i.e., $b_{0}$ to $b_{5}$ and $a_{0}$ to $a_{5}$ are unknown parameters which should be identified.

To identify the unknown parameters of the aforementioned system, first the corresponding output (F.S) should be measured for different inputs (Red, $R$ ) $h_{0}$. Then, through augmentation of the obtained data, a matrix equation can be formed whose left hand eector consist of the outputs and its right-hand is a multiplication of the vector of the coefficients and a regression matrix, as follows: Regression of the following final form: The following formula shows the total shape of the said matrix equation:

$$
\begin{align*}
& \mathrm{Z}=\mathrm{A}(\mathrm{x}) \times \mathrm{B}(\mathrm{y})  \tag{7}\\
& \mathrm{x}=\sqrt{\frac{\mathrm{R}}{\mathrm{~h}_{0}}}  \tag{8}\\
& \mathrm{y}=\operatorname{Red} \tag{9}
\end{align*}
$$



As was mentioned in the previous section, by using of recorded graphs in data logger computer of the finishing mill section, slip forvard factor in finishing mill stands can be measurable. For each load in finishing mill section, thickness of the input load and the relative thickness reduction in each stand is also determined.

Hence, the goal is to find the optimum coefficients $a_{i}$ and $b_{i}$ in the formula $\mathrm{z}=\mathrm{A}(\mathrm{x}) \cdot \mathrm{B}(\mathrm{y})$ by measurin $\mathrm{x}, \mathrm{y}$ and z variables, for some instances.

## APPRQXIMATION THEORY

Surely of the best method in determining optimum coefficients in an approximate equation is using approximation theory. Based on this method, if by formula $A\left(x_{k}\right) \times B\left(y_{k}\right)$, the amount of $z_{k}$ is approximated, then the sum of error squares among measured valued is described as follows:

$$
\begin{equation*}
\mathrm{SSE}=\sum_{\mathrm{k}}\left(\mathrm{Z}_{\mathrm{k}}-\mathrm{A}\left(\mathrm{x}_{\mathrm{k}}\right) \cdot \mathrm{B}\left(\mathrm{y}_{\mathrm{k}}\right)\right)^{2} \tag{10}
\end{equation*}
$$

To minimize SSE, the following nonlinear system of equations should be calculated:

$$
\left\{\begin{array}{l}
\frac{\partial S S E}{\partial a_{i}}=0  \tag{11}\\
\frac{\partial S S E}{\partial b_{j}}=0
\end{array}\right.
$$

This decimal system includes nonlinear equation $i+j$. Since in this study $A(x)$ and $B(y)$ are both polynomials with 6 coefficients, therefore, a system of 12 equations and 12 unknown ( $a_{i}$ and $b_{j}$ ) is obtained. The following formula shows one of equations:

$$
\begin{align*}
& \frac{\partial S S E}{\partial \mathrm{a}_{0}}=0 \quad \Rightarrow \quad \sum_{\mathrm{k}} 2 *\left[\mathrm{~A}\left(\mathrm{x}_{\mathrm{k}}\right) * \mathrm{~B}\left(\mathrm{y}_{\mathrm{k}}\right)-\mathrm{Z}_{\mathrm{k}}\right] * \mathrm{~B}\left(\mathrm{y}_{\mathrm{k}}\right)=0  \tag{12}\\
& \left\{\begin{array}{lll}
A(x)=\sum_{i=1}^{5} a_{i} x^{i} & , & x=\sqrt{\frac{R}{h_{0}}} \\
B(y)=\sum_{j=1}^{5} b_{j} y^{j} & , & y=\operatorname{Red}
\end{array}\right.
\end{align*}
$$

After setting up the above system of equations, a method to solve this system should be chosen which is discussed in the next section.

## THE NEWTON METHOD OF CALCULATING A NONLINEAR SYSTEM OF EQUATIONS

There are several methods to calculate a nonlinear system of equations among them is Newton, Broyden and Steepest Descent methods.

All of the three are iterative method in the present study, and three methods were examined and it was observed that the Newton method is the best choice. This is because Broyden method because of its nature appeared as a divergent method for this system and Steepest Descant method was convergent but, through spending more time. Therefore, the method used is the Newton method whose algorithm described below:

1. A primary guess such as S0is designated. Then, through the following repetitive formula this guess is modified until it reaches the desired solution

$$
\mathrm{S}_{\mathrm{k}+1}=\mathrm{S}_{\mathrm{k}}-\left[\mathrm{J}\left(\mathrm{~S}_{\mathrm{k}}\right)\right]^{-1} * \mathrm{~F}\left(\mathrm{~S}_{\mathrm{k}}\right)
$$

where the operator * represents a matrix product and $J$ is Jacobian matrix:


$$
\begin{equation*}
\Delta_{k}=S_{k+1}-S_{k}=-\left[J\left(S_{k}\right)\right]^{-1} * F\left(S_{k}\right) \tag{13}
\end{equation*}
$$

Therefore,

$$
\begin{equation*}
\left[J\left(S_{k}\right)\right] * \Delta_{k}=-\left[J\left(S_{k}\right)\right] *\left[J\left(S_{k}\right)\right]^{-1} * F\left(S_{k}\right) \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
\left[J\left(S_{k}\right)\right] * \Delta_{k}=-F\left(S_{k}\right) \tag{15}
\end{equation*}
$$

2. $k=0$ is set.
3. By knowing $S_{k}$ the values of $J\left(S_{k}\right)$ and $F\left(S_{k}\right)$ are calculated.
4. Formula (15), with $F\left(S_{k}\right), J\left(S_{k}\right)$ being known, becomes a linear equation system with $\Delta_{\mathrm{k}}$ unknown.
Through using a suitable numerical method which is explained in the next section thoroughly, the system can be solved.
5. The new amount of S is obtained throug $S_{k+1}=S_{k}+\Delta_{k}$
6. If $\left\|\Delta_{k}\right\|_{2}$ is small enough, in other words, if $\left\|\Delta_{k}\right\|_{2}<\varepsilon$; then, $S_{k+\infty}$ is the desired solution and algorithm is finished. Otherwise, $k=k+1$ is set and we go back to step
It is of important to note that $\left\|\Delta_{k}\right\|_{2}$ is 2-norm of the matrix $\Delta_{k}$.
In this method after some iteration, in the case of being convergent, the optimum coefficients will be obtained.

## GAUSSAN ELIMINATION WITH SCALED PARTIAL PIVOTING FOR CALCULATING A SYSTEEM OF LINEAR EQUATIONS:

To solve a system of linear equations, (15), Gausstian elimination with scaled partial pivoting which is a direct method can be used in Nevton algorithm. Algorithm of Gaussian elimination method is as follows:
To calculate a linear system of equations:


First, we form the augmented matrix

$$
\left[\begin{array}{cccc|c}
J_{1,1} & J_{1,2} & \ldots & J_{1,12} & J_{1,13}  \tag{17}\\
J_{2,1} & J_{2,2} & \ldots & J_{2,12} & J_{2,13} \\
\vdots & \ddots & \ddots & \vdots & \vdots \\
J_{12,1} & J_{12,2} & \ldots & J_{12,12} & J_{12,13}
\end{array}\right]
$$

And then act as follows:
If $J_{i, i} \neq 0$ we go to section 4, but if $J_{i, i}=0$ we go to section3. Suppose that p is an integral number in away that $J_{i, i}=J_{i+1, i}=\cdots=J_{p-1, i}=0$ but $J_{p, i} \neq 0$. If there is such a P, we change the lines I and P and go to section 4. If there is no such P , the system doesn't have a unique answer and the process stops.

1. Set $i=1$.
2. If $J_{i, i} \neq 0$, go to step 4, but if $J_{i, i}=0$, go to step 3 .

Suppose that p is an integer number in a way that $J_{i, i}=J_{i+1, i}=\cdots=J_{p-1, i}=0$, but $J_{p, i} \neq 0$. If there is such a $p$, we change the rows $i$ and $p$ and go to step 4 . If there is no such $p$, the system does not have a unique solution and the process stop.
3. Suppose that $j=i+1, i+2, \cdots, 12$, we subtract the product of $m_{j i}$ and $i$ th row from the $j$ th row and replace it in the $j$ th row. In other words $\left(E_{j}-m_{j i} E_{i}\right) \rightarrow\left(E_{j}\right)$.
4. Set $i=i+1$.
5. If $i<12$, we go to step 2 , but if $i=12$, we go to step 7 .
6. If $J_{n, n}=0$ the system does not have a unique solution and the process stops. But if $J_{n, n} \neq 0$, then:
6.1. Set $\delta_{12}=\frac{J_{12,13}}{J_{12,12}}$
6.2. We calculate the amount of $\delta_{i}=$
6.3. The process will terminate,

## RESULTS, DISCUSSION AND CONCLUSION

In the previous section, first, the agorithm of calculating slip forward factor in ultimate rolling section was explained.
In this study a friendly computer software called FSLIP is provided, to perform the algorithm. The merits and capabilities of FSLIP computer model are as follows:

1. The software has been made in windows and has the capability of being installed on computers equipped with any windows.
2. Installation of this software is done simply and very quickly and it will not occupy much space of the computer memory. Also, to install and startup this software on the computer no extra program is needed.
3. This software has been provided with graphical menus. Furthermore, the way of using it is very simple so that each and every user who has little information about computer can use it.
4. This software has help files that enable user to refer to them in different stages of using it and receive lines to use it correctly.
5. In compiling this computer model, the effort was to use methods and algorithms that in addition to accuracy decrease the program's performance time to the least.
In the following table which is a sample of representation of FSLIP program results, the values of the thickness reduction (He [amm] (REDUC [\%]). Radius of rollers (Radius[mm]) and measured value of slip forward factor in rolling stands (Measured FSLF [\%]) and estimated slip forward factor using the new coefficient (New FSLF[\%]) and the errors are presented.

Table1. Sample measured and calculated data

| He | REDUC | RADUCTION | FSLF [\%] |  |  | FSLF ERROR |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $[\mathrm{mm}]$ | $[\%]$ | $[\mathrm{mm}]$ | Given | Original | Estimated | Original | Estimated |
| 37.00 | 33.55 | 389.25 | 6.34 | 7.29 | 6.73 | -0.95 | -0.39 |
| 24.59 | 35.49 | 376.65 | 7.38 | 8.12 | 6.71 | -0.74 | 0.67 |
| 15.86 | 32.86 | 371.00 | 6.10 | 8.02 | 6.06 | -1.92 | 0.04 |
| 10.65 | 29.89 | 364.30 | 7.27 | 7.63 | 6.29 | -0.36 | 0.98 |
| 7.47 | 21.22 | 347.75 | 7.23 | 5.55 | 6.89 | 1.68 | 0.34 |
| 5.88 | 19.71 | 351.45 | 7.93 | 5.24 | 7.59 | 2.69 | 0.34 |
| 37.00 | 30.34 | 389.25 | 7.74 | 6.65 | 6.39 | 1.09 | 1.35 |
| 25.77 | 37.18 | 376.65 | 6.58 | 8.37 | 7.01 | -1.79 | -0.43 |
| 16.19 | 31.48 | 371.00 | 4.55 | 7.69 | 5.92 | -3.14 | -1.37 |
| 11.09 | 26.15 | 364.30 | 6.36 | 6.66 | 5.96 | -0.30 | 0.40 |
| 8.19 | 22.30 | 347.30 | 5.30 | 5.79 | 6.59 | -0.49 | -1.29 |
| 6.37 | 15.38 | 351.45 | 8.43 | 4.07 | 7.24 | 4.36 | 1.19 |
| 37.00 | 34.17 | 389.25 | 7.51 | 7.41 | 6.80 | 0.10 | 0.71 |
| 24.36 | 36.90 | 376.65 | 7.45 | 8.39 | 6.90 | -0.94 | 0.55 |
| 15.37 | 32.60 | 371.00 | 5.50 | 7.99 | 6.03 | -2.49 | -0.53 |
| 10.36 | 27.90 | 364.30 | 5.95 | 7.15 | 6.23 | -1.20 | -0.28 |
| 7.47 | 22.50 | 347.75 | 8.12 | 5.89 | 6.93 | 2.23 | 1.19 |
| 5.79 | 18.56 | 351.45 | 8.18 | 4.94 | 7.57 | 3.24 | 0.61 |
| 34.00 | 36.51 | 389.25 | 7.21 | 7.94 | 7.12 | -0.73 | 0.09 |
| 21.59 | 38.09 | 376.65 | 6.45 | 8.75 | 6.93 | -2.30 | -0.48 |
| 13.36 | 33.15 | 371.00 | 5.98 | 8.23 | 6.19 | -2.25 | -0.21 |
| 8.93 | 27.55 | 364.30 | 7.77 | 7.15 | 6.70 | 0.62 | 1.07 |
| 6.47 | 22.69 | 347.75 | 6.22 | 6.00 | 7.44 | 0.22 | -1.22 |
| 5.00 | 18.16 | 351.45 | 5.73 | 4.88 | 7.63 | 0.85 | -1.90 |
| 21.16 | 39.27 | 376.65 | 7.68 | 8.97 | 7.08 | -1.29 | 0.60 |
| 12.85 | 36.87 | 371.00 | 7.14 | 8.94 | 6.64 | -1.80 | 0.50 |
| 8.18 | 27.91 | 364.30 | 6.33 | 7.29 | 7.05 | -0.96 | -0.72 |
| 4.55 | 18.05 | 35145 | 814 | 4.88 | 7.48 | 3.26 | 0.66 |
| 37.00 | 29.39 | 389.25 | 6.94 | 6.45 | 6.32 | 0.49 | 0.62 |
| 26.12 | 37.27 | 376.65 | 7.03 | 8.37 | 7.04 | -1.34 | -0.01 |
| 16.39 | 30.72 | 371.00 | 4.91 | 7.51 | 5.86 | -2.60 | -0.95 |
| 11.35 | 30.02 | 364.30 | 6.43 | 7.62 | 6.15 | -1.19 | 0.28 |
| 7.94 | 22.47 | 347.75 | 8.22 | 5.85 | 6.71 | 2.37 | 1.51 |
| 6.16 | 24.41 | 35145 | 5.76 | 6.49 | 7.69 | -0.73 | -1.93 |
| 34.00 | 39.68 | 389.25 | 7.25 | 8.46 | 7.61 | -1.21 | -0.36 |
| 20.51 | 42.92 | 376.65 | 7.60 | 9.55 | 7.51 | -1.95 | 0.09 |

The obtained new slip factor coefficients based on the above data are as follows:

| $i$ | $a_{i}$ | $b_{i}$ |
| :---: | :---: | :---: |
| 0 | -5.0038809208431 | 0.1262496837933 |
| 1 | 5.6637730611968 | -1.2915759281692 |
| 2 | -2.0755476511603 | 11.3943906564029 |
| 3 | 0.3548131355540 | -47.7452126449864 |
| 4 | -0.0283239316792 | 96.7116392325892 |
| 5 | 0.0008525567105 | -74.0299740147414 |

Fig. 3 and Fig. 4 illustrate the graphs of the calculated FS factor by using the original and new coefficients and their differences with the measured data (errors), respectively.


Fig. 3. Slip forward factor by the original formula, the new estimated formula and the measured


Fig. 4. Differences between the measured data and calculated slip forward factors by the new and origiñal formulas.
Also the errors are:
Sum of Square Error: Given vs. Estimated $=62.291$, Given vs. Original $=322.7141$
Sum of Absolute Error: Given-vs. Estimated $=0.67696$, Given vs. Original $=1.6159$

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